state equation by an ICS to eliminate the bias or cancel the incorrect gain, thereby accommodating the sensor faults and allowing the closed-loop system to run as if unimpaired.

Several issues remain to be investigated once noise is included in the system: the effect on threshold values against which residuals are compared, the ability of an appropriate parameter identification technique to provide bias-free estimates, and the minimum number of samples included in the moving window for convergence of the estimates.

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On-Line Robust Stabilizer

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I. Introduction

THIS Note presents an adaptive robust discrete solution for a stabilization problem. As an application of this solution, a stability augmentation system (SAS) for an aircraft is presented.

The general scheme of this system is given in Fig. 1. Throughout this Note two hypotheses are assumed: 1) full information about the state of the system ($y_k = x_k$) and 2) that the identification block gives the best fit of the linearized time-varying nonlinear system (i.e., $x_{k+1} = A_k x_k + B_k u_k$) (see, e.g., Ref. 1).

The sample time of the control loop is much less than that of the identification and optimization loop. This allows on-line identification and optimization procedures.

A convenient criterion to be optimized is sought such that the control u_k will be given by a linear state feedback: $u_k = F_k x_k$. This criterion will include two terms: one involving the performance requirements and the other involving the stability robustness:

$$C_{PR} = \lambda C_P + C_R$$

where λ is a weighting parameter.

II. Optimization Problem

The feedback matrix F_k will be given by a Riccati equation, but instead of a classical discrete algebraic Riccati equation (DARE),

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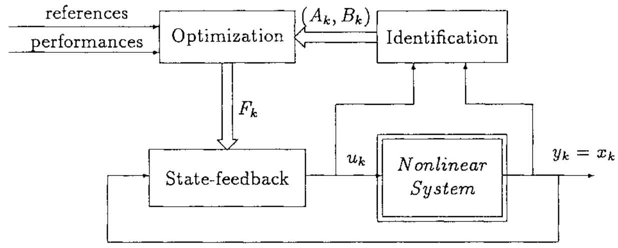

Fig. 1 General scheme.

a modified DARE will be used such that the eigenvalues of the stable closed-loop matrix are to be within some disk included in the unit disk (according to Ref. 2 this is called the D-pole assignment problem). Let us consider the disk $D(\alpha, r)$ of center α and radius r in the complex plane such that α is a real number and $|\alpha| + r \leq 1$ [i.e., $D(\alpha, r) \subset D(0, 1)$]. In order to state the criterion, the following DARE is considered:

$$\tilde{A}_k^T K_k \tilde{A}_k - K_k - \tilde{A}_k^T K_k B_k \left(R + B_k^T K_k B_k\right)^{-1} \times B_k^T K_k \tilde{A}_k + Q = 0 \tag{1}$$

where $\tilde{A}_k = (A_k - \alpha I)/r$ is the modified A matrix and $Q, R > 0$ are positive matrices related to the quadratic cost:

$$I = \sum_{j \geq k} \left(x_j^T Q_k x_j + u_j^T R u_j\right) \tag{2}$$

with

$$Q_k = Q + \tilde{A}_k^T K_k \tilde{A}_k - A_k^T K_k A_k + A_k^T K_k B_k \left(R + B_k^T K_k B_k\right)^{-1} \times B_k^T K_k A_k - \tilde{A}_k^T K_k B_k \left(R + B_k^T K_k B_k\right)^{-1} B_k^T K_k \tilde{A}_k \tag{3}$$

and the linear dynamics $x_{j+1} = A_k x_j + B_k u_j$, $j \geq k$.

It is known that there exists a unique stabilizable solution $K_k = K_k^T > 0$ of Eq. (1).[3] Let us set

$$F_k = -r\left(R + B_k^T K_k B_k\right)^{-1} B_k^T K_k \tilde{A}_k \tag{4}$$

and

$$A_{s,k} = A_k + B_k F_k \tag{5}$$

Since $\Lambda(\tilde{A}_k + B_k F_k / r) \subset D(0, 1)$, one can obtain

$$\Lambda(A_{s,k}) \subset D(\alpha, r) \tag{6}$$

where $\Lambda(\cdot)$ denotes the eigenvalues set of the matrix.

Now the criterion to be minimized can be stated as follows:

Performance part:

$$C_P = x_k^T K_k x_k \tag{7}$$

Stability robustness part:

$$C_R = x_k^T A_{s,k}^T A_{s,k} x_k = \|A_{s,k} x_k\|^2 \tag{8}$$

The first part of the criterion is related to the quadratic cost (2) of the dynamics and the matrices Q and R are chosen to fulfil the performance requirements.

The second part gives an ∞-norm bound along the trajectory: the goal is to minimize not the H_∞-norm of A_s (which is $\max_{\|x\|=1} \|A_s x\|$) but just the $\|A_s x\|$ along the trajectory; since it is not known a priori the trajectory, in an on-line solution only $\|A_s x\|$ with x taken to be the actual state ($x = x_k$) is to be minimized. The form of C_R is suggested by the constrained stability measure introduced in Ref. 4, taking for G the real trajectory of our system and also $P = I$ in the formula (17) of the cited paper.

Putting these two expressions together in the criterion, one can obtain a trade-off between performance and stability. Indeed, whereas to minimize the first part means to approach the unit circle [this is the solution of the linear quadratic (LQ) problem], the second part leads to minimizing the radius of the circle around the same center (depending on x_k). The optimum of the criterion (which is the minimum) gives us the solution.

The criterion is

$$C_{PR} = x_k^T \left(\lambda K_k + A_{s,k}^T A_{s,k}\right) x_k \tag{9}$$

where K_k is given by Eq. (1) and $A_{s,k}$ by Eq. (5) [and (4)]. The optimization problem is then

$$\min_{\substack{\alpha,r \\ r\geq 0, |\alpha|+r\leq 1}} \quad C_{PR}(x_k,\ A_k,\ B_k;\ \alpha,\ r) \tag{10}$$

III. Algorithm

Assuming that the model (A_k, B_k) is slowly time varying, an online algorithm to solve Eq. (10) is proposed. The idea is not to solve Eq. (1) directly but to approach recursively the solution using the fixed point method.

To initialize the algorithm, it is supposed that an initial estimation (A_0, B_0) of the model and K_0, which is the exact solution of DARE (1) and minimizes Eq. (10), are known. Then, at the kth step:

Given x_k, A_k, B_k, K_{k-1} (and Q, R).

Set $U = (R + B_k^T K_{k-1} B_k)^{-1} B_k^T K_{k-1}$.

Search the minimum:

$$\min_{\substack{\alpha,r \\ r\geq 0, |\alpha|+r\leq 1}} \quad \left[\tilde{C} = x_k^T \left(\lambda K_k + A_{s,k}^T A_{s,k}\right) x_k\right]$$

Subject to the relations

$$\tilde{A}_k = (A_k - \alpha I)/r$$

$$F_k = -rU\tilde{A}_k$$

$$A_{s,k} = A_k + B_k F_k$$

$$K_k = \tilde{A}_k K_{k-1}(A_{s,k} - \alpha I)/r + Q$$

using a gradient procedure with convex restrictions (given by α and r) and variable step.

The control is $u_k = \hat{F}_k x_k$.

IV. Simulation Results

The longitudinal dynamics of the airplane is given in Ref. 5 and has the form

$$\begin{aligned}
\frac{dv}{dt} &= -p\frac{S}{m}(C_D - C_T \cos\alpha) - g \sin\gamma \\
\frac{d\gamma}{dt} &= p\frac{S}{mv}(C_L + C_T \sin\alpha) - g\frac{\cos\gamma}{v} \\
\frac{dq}{dt} &= p\frac{SC}{I_y}C_M \\
\frac{d\theta}{dt} &= q \\
\frac{dH}{dt} &= v \sin\gamma
\end{aligned} \tag{11}$$

where v is the airspeed, p is the dynamic pressure, S is the wing area, C_D is the drag coefficient, C_L is the lift coefficient, C_M is the

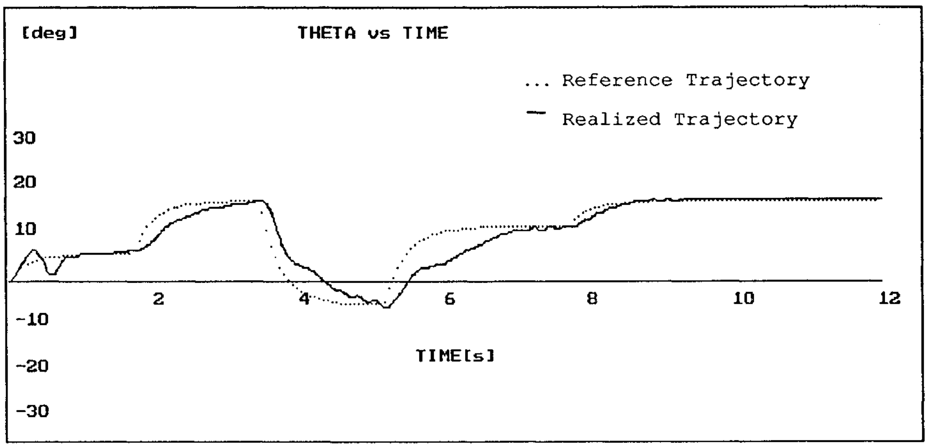

Fig. 2 Reference and realized trajectories.

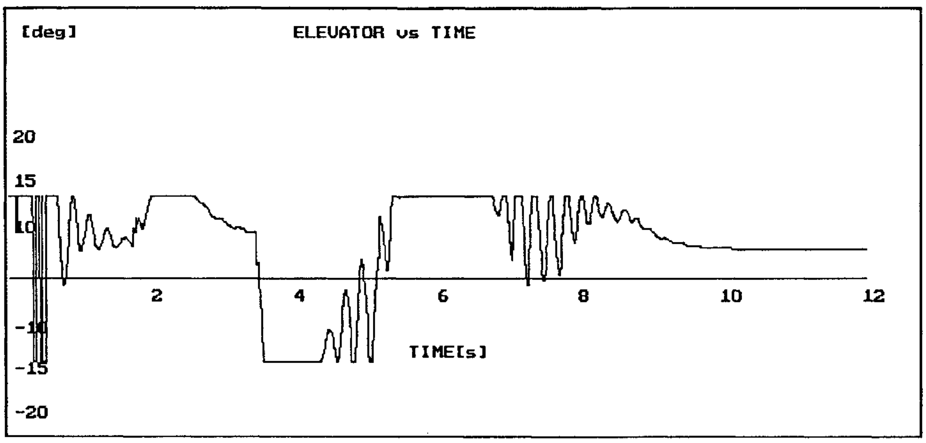

Fig. 3 Control vs time.

moment coefficient, C is the chord, I_y is the moment of inertia, γ is the flight-path angle, q is the pitch rate, α is the angle of attack, θ is the attitude angle, and H is the altitude.

The problem is to bring the system to follow the programmed trajectory given by the attitude angle $\theta_p(t)$. Then the model state x_k will contain the tracking error and its derivatives with respect to t. This nonlinear model can be linearized around the actual error and can be brought into a controllable and observable parametrized state space model of the form

$$x(k+1) = A(\xi_k)x(k) + B(\xi_k)u(k), \qquad x(0) = x_0$$

where the control u is given by the elevator command and linearly modifies the moment, lift, and drag coefficients C_M, C_L, C_D. The actual values of parameters A and B are obtained from the identification block. The open-loop model is unstable. The dimension of the model has been chosen $n = 3$. The state of the system is then $x = (e = \theta - \theta_p, \dot{e}, \ddot{e})$. In the criterion (2) we have taken $Q = I_3$, $R = 2$ and the weighting parameter $\lambda = 0.02$.

In Fig. 2 are drawn both a reference trajectory θ_p, with a dotted line, and the attitude angle obtained, with a continuous line. In Fig. 3 is represented the control variable that is the elevator deflection. In the first second the identification has caused a lot of oscillations for the computed control. Moreover, it has been simulated with gust as in Ref 6.

One can see that the system tracks well the reference trajectory (this achieves the performance requirement). The presence of gust does not generate instability (this suggests the achievement of the robustness properties). The value of λ has been chosen as above in order to obtain the same order for the two terms C_P and C_R in the criterion (9).

As a conclusion, the controller tracks the desired trajectory and achieves a trade-off between the performance requirements and the robustness of the stability.

V. Conclusions

In this work an on-line solution for a robust stabilization problem is presented. As an application, a SAS for an aircraft is realized and the results are discussed. The idea behind the robustness criterion is to place the poles of the closed-loop system within some disk included in the unit disk such that the criterion is minimized. The freedom degrees are in this case the radius and the position of the center of the disk. Since α must be a real number, one can see that only symmetric disks with respect to the real axis are allowed.

The criterion to be minimized has two parts [see Eq. (9)], one involving some performance requirements, related to the stabilizable solution of a certain modified DARE [Eq. (1)], and another giving the robustness of the stabilized system. We stress that the stability robustness part [C_R from Eq. (8)] is given by the norm of a state vector and not by the norm of a transfer matrix. This part of the criterion has been chosen like this because of the adaptivity of the solution. The algorithm presented here to solve the optimization problem uses the fixed-point method in order to solve the modified DARE. This fact allows an on-line implementation with a faster adaptivity.

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Matched Asymptotic Expansion Solutions for an Ablating Hypervelocity Projectile

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Introduction

PREVIOUS studies have demonstrated the importance of ablative mass losses of direct launch system projectiles during atmospheric ascent.[1,2] In this Note matched asymptotic expansion solutions[3,4] are obtained which represent the trajectories of such projectiles and incorporate a simple ablation model. Ablative mass losses are parametrically tied to changes in projectile cross section and so are strongly coupled to the projectile flight dynamics. The ablation model assumes that a fraction of the kinetic heating is directed to vaporize the projectile surface. More complex models are available which include heat transfer blocking due to the ablating surface material. However, these effects are found to be small. The solutions obtained give uniformly valid representations of the projectile trajectory from launch tube exit to exoatmospheric Keplerian motion. Such solutions are of interest for the rapid evaluation of direct launch system performance and for other ablative problems, such as kinetic energy antiballistic missile projectiles.

Projectile Dynamics

A generic trajectory geometry will be considered as illustrated in Fig. 1. It will be assumed that there are no transverse forces so that the projectile motion is purely planar. In a geocentric inertial frame the dynamical equations may be written as

$$\frac{dv}{dt} = -\frac{1}{2}\rho C_D \frac{S}{m} v^2 - \frac{\mu}{r^2}\sin\gamma \tag{1a}$$

$$v\frac{d\gamma}{dt} = -\left\{\frac{v^2}{r} - \frac{\mu}{r^2}\right\}\cos\gamma \tag{1b}$$

$$\frac{dr}{dt} = v\sin\gamma \tag{1c}$$

$$\frac{dm}{dt} = -\frac{1}{2}\frac{\kappa}{q}\rho v^3 \tag{1d}$$

where the projectile is characterized by mass m, cross section S, and drag coefficient C_D. The ablative mass loss is modeled by Eq. (1d) by assuming a fraction κ of the kinetic heating is directed to vaporize the projectile surface with specific heat of vaporization q (Ref. 5). Additional terms may be included to model the reduction of heat transfer through the boundary layer. However, for turbulent boundary flow these terms are found to be small.

As the projectile ablates mass, its cross section S will also change. The change in cross section will be described through the shaping parameter α by a mass power law, viz.,

$$S = S_0\{m/m_0\}^\alpha \tag{2}$$

where the projectile has initial mass m_0 and initial cross section S_0. Projectile geometries may be modeled by a suitable choice of shaping parameter. If the ablating projectile remains self-similar then $\alpha = 2/3$, whereas for a cylinder ablating from one end $\alpha = 0$. In

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